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Procedia - Social and Behavioral Sciences 54 (2012) 685 – 695

Procedia
Social and Behavioral Sciences

EWGT 2012

15th meeting of the EURO Working Group on Transportation

A metaheuristic dynamic traffic assignment model for O-D matrix estimation using aggregate data

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Abstract

For a static/dynamic O-D matrix estimation, usually, the basic required information is a starting estimation of O-D matrix and a set of traffic counts. In the era of the Intelligent Transportation Systems, a dynamic estimation of traffic demand has become a crucial issue. Different Dynamic Traffic Assignment (DTA) models have been proposed, used also for O-D matrices estimation. This paper presents a dynamic O-D demand estimator, using a novel simulation-based DTA algorithm. The core of the proposed algorithm is a mesoscopic dynamic network loading model used in conjunction with a Bee Colony Optimization (BCO). The BCO is capable to solve high level combinatorial problems with fast convergence performances, allowing to overcome classical demand-flow relationships drawbacks.

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Keywords: Metaheuristic; Dynamic assignment; O-D matrix estimation; Traffic counts; Bee colony optimization.

1. Introduction

The estimation of traffic flows on a network provides to transportation planners important information for traffic network design and control. For this reason, many researchers have directed their studies toward the definition of sophisticated and effective traffic assignment models. Nevertheless, reliability and effectiveness of traffic assignment models depend on other important elements such as Origin - Destination (O-D) travel demand, that is the core input of traffic assignment models. Effective and theoretically consistent methodologies have been proposed to estimate O-D matrices by using aggregate data such as traffic counts and/or O-D demand counts, since the use of information derived from traffic counts is very attractive because they are cheap, easy to obtain as

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automated, and immediate data to collect. The specific literature in this field is very broad and different classes of estimators have been investigated in order to analytically specify and solve this estimation problem.

For a static/dynamic O-D matrix estimation, the basic required information depend on the traffic assignment model, and usually are a starting estimate of the static/dynamic O-D matrix(es), i.e. the target O-D demand(s), and a set of aggregate data such as traffic counts or speed measures observed on the links of the considered network (Bera and Rao, 2011; Frederix et al. 2011).

In the era of the Intelligent Transportation Systems (ITS), for a real-time management of transportation systems, a dynamic estimation of traffic demand has become more important than the static estimation case (Ashok and Ben-Akiva, 1993 and 2002) that is more suitable for tactical or strategic planning. Given the time dependence of traffic flows, Dynamic Traffic Assignment (DTA) models are necessary to predict traffic patterns as a result of demand/supply interactions, in order to implement information schemes of route guidance.

Different DTA models have been proposed (Merchant and Nemhauser 1978a and 1978b; Friesz et al. 1989; Balijepalli et al., 2007), used also for O-D matrices estimation (Van der Zijpp, 1997; Van der Zijpp and Lindveld, 2001; Flötteröd et al., 2011). Generally, they can be classified into two categories: analytical models and simulation-based models (Juran et al. 2009). In the analytical DTA models the ability to reproduce realistic dynamics of traffic is unsatisfactory especially for congested networks. Conversely, the simulation-based approaches have some benefits (Mahmassani and Peeta, 1993; Jayakrishnan et al. 1995) and are developed mainly for practical applications. This kind of models has been developed in past decades for the design of a variety of traffic network, for planning and operation problems (Ben-Akiva et al. 2001). They are based on sophisticated algorithms and detailed macroscopic, microscopic and mesoscopic simulation techniques to estimate current network performance, predict future conditions and generate traffic guidance. Generally, mesoscopic simulation models achieve a balance between computational constraints in real-time applications and accuracy in description of network conditions with an adequate level of detail.

This paper presents an estimation of flows of dynamic O-D demand, using a new simulation-based DTA algorithm. In order to solve the O-D matrices estimation, we have used a Generalized Least Square estimator, based on traffic measurements. Dynamic estimates of O-D matrices can be obtained in a single-level minimization. The objective function consists of weighted Euclidean distances between the target O-D dynamic demands and measured traffic flows, from one side; from the other side, of the estimates of O-D dynamic demands and link flows resulting from the DTA.

The core of the proposed DTA algorithm is a mesoscopic dynamic network loading model (DNL). In particular, in order to calculate the movement of vehicles on links, we use a time-discrete mesoscopic model, which assumes that vehicles are grouped into discrete packets, arranged according to their entry time. Vehicles entering or leaving a link are respectively added or subtracted in the appropriate time intervals to produce new densities and flows estimates. The DNL model is used in conjunction with a Bee Colony Optimization (BCO) to solve the DTA problem. The BCO metaheuristic represents a new direction in the field of Swarm Intelligence (SI). Generally, SI systems are made of a population of agents that individually do not show an “intelligent” behaviour. However, local interactions with each other and with their environment lead to an “intelligent” global behaviour, unknown to the individual agents. As well as bee colonies, examples of SI include ant colonies, bird flocking, animal herding, bacterial growth, and fish schooling. Several scientists have studied bee colonies in nature (Gould, 1987; Dukas and Visscher, 1994; Waddington et al., 1998; Williams and Thompson, 1998). Not so long ago the first application of bee colony algorithm to combinatorial problems in Transportation Engineering has been proposed (Lučić and Teodorović, 2001). Due to its inherent interest, this approach has been later followed by many scientists (Karaboğa and Başturk, 2007 and 2008; Teodorović and Dell’Orco, 2008).

The BCO is capable to solve high level combinatorial problems with fast convergence performances, allowing to overcome classical demand - flow relationships drawbacks (such as the calibration of path choice coefficients). Due to the dynamic nature of the model, the BCO evaluates all the possible paths to destination from every node reached by the traffic flow and finds the best paths at each time interval.

On-going step is the application of the proposed estimation procedure to a test network and to a real one. In order to evaluate the first outcomes, “true” O-D matrices have been set up. Target O-D demands and measured flows vectors were obtained from a normal distribution, having the true values as average values. The evaluation of the performances of the proposed method is carried out using mean square error between the true and the estimated values.

2. Statement of the problem (O-D Matrix Estimation Model)

The general O-D demand estimation problem is to find an estimate of O-D demand matrix by effectively utilizing traffic flow observations and other available information. The review focuses on different estimation and different assumptions for the O-D demand estimation problem under different information sources. Existing O-D demand estimation models belong to two major categories: static models or dynamic models. Assuming constant trip desires over the estimation horizon, static O-D demand estimation models estimate a static O-D demand table based on daily or hourly average traffic counts. To realistically represent traffic formation and congestion on the traffic network, dynamic models utilize time-varying traffic flow observations to estimate traffic demand that varies over time. The most general form to solve the estimation of O-D matrix using traffic counts can be formulated as an optimization problem (Cascetta, 1986; Cascetta and Nguyen, 1988; Yang et al., 2001):

$$\mathbf{d}^* = \arg \min_{\mathbf{d} \in S_d} [F_1(\mathbf{d}, \bar{\mathbf{d}}) + F_2(\mathbf{f}, \hat{\mathbf{f}})] \quad (1)$$

where: S_d is the feasible solution set to the problem; F_1 and F_2 are, respectively, measures of the “distance” between the starting estimate $\bar{\mathbf{d}}$ (target demand vector) and the unknown vector \mathbf{d} and between estimated link flows vector \mathbf{f} and traffic counts vector $\hat{\mathbf{f}}$.

The distances F_1 and F_2 can be defined following different statistical approaches and assumptions. The most used estimator is the Generalized Least Square (GLS) method.

Moving towards the dynamic framework, an extension of the static O-D correction procedure for obtaining time-varying O-D flows using time-varying traffic counts was provided by Cascetta et al. (1993), who proposed two different dynamic estimators, i.e. a simultaneous and a sequential estimator.

The simultaneous estimator jointly estimates all O-D matrices for all time slices using the whole set of traffic counts, with a specification directly derived from estimator.

The sequential estimator is based on the estimation at each interval of the O-D demand \mathbf{d}_t for that single interval h , expressing traffic counts of time slice h as a function of \mathbf{d}_h and of the already estimated demand flows of previous intervals ($\hat{\mathbf{d}}_{t_1}; \hat{\mathbf{d}}_{t_2}; \dots$); a general formulation of the corresponding formulation is reported in equation (2) wherein subscript t refers to each time slice $t = [1, \dots, H]$.

$$\mathbf{d}_t^* = \arg \min_{\mathbf{d}_t \geq 0} \left[F_1(\mathbf{d}_t, \hat{\mathbf{d}}_t) + F_2(\mathbf{f}_t(\mathbf{d}_t | \mathbf{d}_{t_1}^*, \dots, \mathbf{d}_{t_{H-1}}^*), \hat{\mathbf{f}}_t) \right] \quad (2)$$

The sequential estimator offers computational advantages, since it reduces a large optimization problem into a number of smaller ones and gives the possibility of using the estimates for an interval as a priori estimates of subsequent intervals. Such aspects have made this approach suitable for the real-time estimation problem, while the simultaneous approach is usually used for off-line estimation, even if it can be shown to be inefficient also for moderate size networks, as reported in Cascetta and Russo (1997), Toledo et al. (2003), and Bierlaire and Crittin (2004).

In this work we propose a sequential bi-level dynamic O-D optimization. On the first level, starting from the problem (2), we have considered the following objective function based on the GLS estimator:

$$\mathbf{d}_{GLS}^* = \arg \min_{\mathbf{d}_{t \geq 0}} \sum_{t=1}^H (\mathbf{d}_t \hat{\mathbf{d}}_t) \mathbf{Z}_t^{-1} (\mathbf{d}_t - \hat{\mathbf{d}}_t)^T + \sum_{t=1}^H (\hat{\mathbf{f}}_t - \mathbf{M} \mathbf{P}_t \mathbf{d}_t) \mathbf{W}_t^{-1} (\hat{\mathbf{f}}_t - \mathbf{M} \mathbf{P}_t \mathbf{d}_t)^T \quad (3a)$$

subject to:

$$\alpha_{p,t}^* = \arg \min \sum_{od \in OD} \sum_{p \in P_{od}} \left[\alpha_{p,t} \sum_{i \in I_p} \tau_i(t) + \sum_{\lambda \in \Lambda_p} B_{n,t} \Delta t \right] \quad (3b)$$

where \mathbf{Z}_t and \mathbf{W}_t are the variance-covariance matrices of the error for the target demand vector and for the link flows, respectively. They represent the weights of the available information that can be interpreted as the level of confidence (or the reliability) in the available starting data. $\mathbf{M} = \{m_{i,p}\}$ is the incidence matrix of links i with paths p ; \mathbf{P}_t is the path choice rates' matrix with entries $\alpha_{p,t}$; $\mathbf{M} \mathbf{P}_t = \{m_{i,p} \alpha_{p,t}\}$ is the assignment matrix; $\mathbf{f} = \mathbf{M} \mathbf{P}_t \mathbf{d}$.

The second level (3b) considers the DTA solved with the Bee Colony Optimization metaheuristic explained in the next section.

3. Simulation-based DTA model

Simulation-based models have been developed in past decades, for a variety of traffic network design, planning and operations management situations. The models use sophisticated algorithms and detailed macroscopic, microscopic, and mesoscopic simulation techniques to estimate current network performance, predict future conditions and generate traffic guidance.

In this work, we have considered a mesoscopic Dynamic Network Loading model to simulate road network. Mesoscopic models try to balance computational burden related to real-time applications, and accuracy required to describe the network conditions with an adequate level of details. Examples of such models include CONTRAM by Leonard et al. (1989); Dynamic Network Assignment for the Management of Information to Travelers (DynaMIT) by Ben-Akiva et al. (2001, 2002); DYNAMIC Network Assignment-Simulation Model for Advanced Road Telematics (DYNASMART) by Mahmassani (2002). More recently, a model with uniformly accelerated discrete packets has been proposed to represent outflow dynamics (Dell'Orco, 2006; Celikoglu and Dell'Orco, 2007; Celikoglu et al., 2009). In these studies, a new model with uniformly accelerated discrete packets and a capacity constraint is proposed to tackle over-saturation phenomena.

3.1. The Dynamic Network Loading Model

To calculate the movement of vehicles on links, we have used a time-discrete mesoscopic model, which assumes that vehicles are grouped in discrete packets, arranged in order of their entry time. Vehicles entering or leaving the link are respectively added or subtracted in the appropriate time intervals to produce new densities and flows estimates. The model has been utilized elsewhere (Dell'Orco, 2006; Celikoglu and Dell'Orco, 2007; Celikoglu et al. 2009), but never in conjunction with a metaheuristic to achieve a Traffic Assignment.

Let P be the set of feasible paths on the network; the set of vehicles, leaving in the same time interval t and following the same path p ($p \in P$) is called a "packet" (t, p) .

We have assumed that:

- vehicles belonging to the same packet are all located at the head of the packet;
- the speed is equal for all packets on the same link;
- the movement of vehicles is uniformly accelerated;
- a relation between speed and density exists and is valid.

On the basis of Greenshields' relationship (1935), the variable $V^i(t)$, speed at time t on link i , can be also calculated as a function of $k_i(t)$, density on link i at time t :

$$V^i(N(t)) = \begin{cases} V_0^i \left(1 - \frac{k^i(t)}{k_{\max}^i} \right) & \text{if } k^i(t) < k_{\max}^i \\ V_{\text{creeping}} & \text{if } k^i(t) \geq k_{\max}^i \end{cases} \quad (4)$$

in which the density on link i with length s occupied by N vehicles at time t is calculated as:

$$k^i(t) = N^i(t)/s^i \quad (5)$$

In case density on link i at time t is greater than maximum allowable density, to avoid the block of flows, we have used a value of $V_{\text{creeping}} = 12$ km/h (6.2 mi/h) as minimum speed. The model given is a fixed-point problem with respect to the variable $V^i(t+\Delta t)$ as described by Dell'Orco (2006). A new value of speed is calculated by the Method of Successive Averages (MSA) as shown in Eq. 6, where $V_r^i(t+\Delta t)$ is the value of speed $V^i(t+\Delta t)$ obtained at iteration r .

$$V_{r+1}^i(t+\Delta t) = \left(\frac{1}{r} \cdot V(N(t+\Delta t)) \right) + \left(\frac{(r-1)}{r} \cdot (V_r^i(t+\Delta t)) \right) \quad (6)$$

The iteration stops when the difference between two consecutive speed values is not greater than the threshold. Then, the current value of acceleration a is calculated as given in Eq. 7, and used as input in successive calculations:

$$a^i(t+\Delta t) = \frac{(V^i(t+\Delta t) - V^i(t))}{\Delta t} \quad (7)$$

Additionally, to handle the queues, we have made the hypothesis that vehicles exceeding the capacities of a link are temporarily stored in a virtual buffer located at the beginning of that link. They are then gradually discharged as soon as the demand falls below the capacity.

3.2. The Bee Colony Optimization approach to DTA problem

The proposed model calculates split factors in a single node d by considering all possible assignments to a diverging link i . The rate of flow assigned to a single path p is calculated through the BCO.

The aim of the optimization process is to solve the problem (8), which considers all paths' costs of all O-D pairs in the network at time t .

$$\min Q_t = \sum_{od \in OD} \sum_{p \in P_{od}} \left[\alpha_{p,t} \sum_{i \in I_p} \tau_i(t) + \sum_{\lambda \in \Lambda_p} N_{\lambda,t} \Delta t \right] \quad (8)$$

where $\tau_i(t)$ is the travel time on link i at time t ; $N_{\lambda,t}$ is the number of vehicles stored in the buffer λ at time t that represents a queue. The optimization problem given by Eq. 8 is solved at each time interval and during simulation within the modeling horizon. Due to dynamic nature of the model, at each time interval the procedure should evaluate all possible paths to destination from every node reached by the traffic flow, facing in this way a "combinatorial explosion". To handle this problem, in this work we have used the Bee Colony (BC) metaheuristic, finding the best paths at each time interval, and thus optimizing the traffic assignment. The agents that we call artificial bees collaborate in order to solve the DTA problem. We create the artificial network shown in the Figure 1. The node represented by the square in the Figure 1 represents the hive. At the beginning of the search process all artificial bees are located in the hive. Bees depart from the hive and fly through the artificial network from the left to the right. Bee's trips are divided into iterations. A bee chooses to visit one artificial node in every iteration which represents the choice of a specific traffic assignment. Bee's paths through the artificial

network represent a sequence of assignments chosen by the bee at time t . We show within every artificial node the physical network that contains all potential paths, previously specified for the given O-D pair. We have determined in advance the number B of bees and the number I of iterations. The following is the pseudo-code of the Bee Colony Optimization metaheuristic which is executed at each time step during simulation.

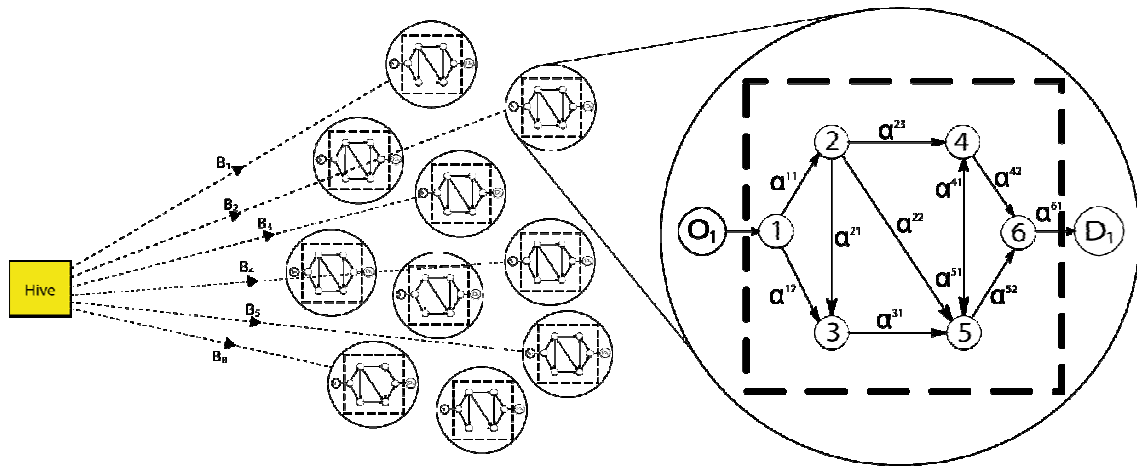


Fig. 1. Forward pass of BCO algorithm

Alternatively, forward and backward passes could be performed until some other stopping condition is satisfied. The possible stopping conditions could be, for example, the maximum total number of forward/backward passes, or the maximum total number of forward/backward passes between two objective function value improvements.

Initialization. Determine the number of bees B , the number of iterations I . Find any feasible solution x of the problem. This solution is the initial best solution.

Set $i = 1$. Until $i = I$, repeat the following steps:

Set $j = 1$. Until $j = m$, repeat the following steps:

Forward pass: Allow bees to fly from the hive and to choose splitting percentages for each network node.

Backward pass: Send all bees back to the hive. Allow bees to exchange information about quality of the assignment chosen and to decide whether to abandon the created solution and become again uncommitted followers, continue to expand the same solution without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the created solution. Set $j = j + 1$.

If the best solution x obtained during the i -th iteration is better than the best-known solution, update the best known solution ($x := x_i$).

Set $i := i + 1$.

4. Sioux Falls City test problem and experimental results

The BCO-based dynamic O-D matrix estimation model has been tested on the Sioux Falls City network shown in Figure 2. This network comprises 24 nodes and 76 links.

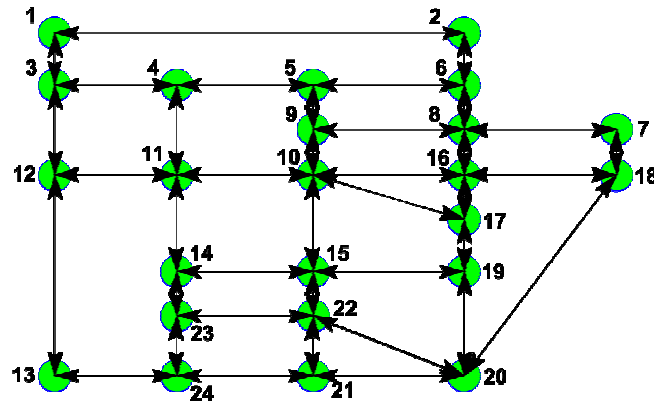


Fig. 2. Sioux Falls City network

We have considered a scenario with 8 O-D pairs and a “true” demand as described in Table 1 in a simulation period of 2 hours. Simulation step interval has been set to 20 seconds.

Table 1. Considered O-D pairs and relative demand flows

[O, D, t(h)]	Demand (vehic/min)	[O, D, t(h)]	Demand (vehic/min)
[1,20,0]	25	[20,1,0]	25
[1,20,1]	40	[20,1,1]	40
[1,24,0]	20	[24,1,0]	20
[1,24,1]	35	[24,1,1]	35
[7,20,0]	15	[20,7,0]	15
[7,20,1]	30	[20,7,1]	30
[7,24,0]	15	[24,7,0]	15
[7,24,1]	30	[24,7,1]	30

Tests have been carried out starting from the simulation of the network in order to obtain link flow counts. Afterwards, we have considered a subset of simulated link flows on the basis of their influence on the overall network dynamics. In the next step, we have applied a white Gaussian noise to link flows and demand. Link flow counts have been perturbed with variation coefficient equal to 0.05, because we give high reliability to traffic counts. Instead, demand has been perturbed with three different variation coefficients equal to 0.2, 0.5 and 0.7 to obtain historical (target) demands. These three experiments have been set up to evaluate the effectiveness of our model in the case of very inaccurate historical demand data. Additionally, we have set up the problem (3a) considering \mathbf{Z}_t as variance matrix of the target demand. Since we give much more reliability to link flow counts,

W_t has been considered as a matrix with unitary elements, in order to give more significance to flows' relative error.

4.1. Results

At the end of each optimization procedure we have obtained the estimated O-D data. These estimated values have been compared with historical ones and the “true” demand. Experimental results have been carried out considering correction performances through the following three indicators:

$$MSE_{od}^{true} = \frac{1}{n_{od}} \sum_{od} (d_{od}^{true} - d_{od}^*)^2 \quad (9)$$

$$MSPE_{od}^{true} = \frac{1}{n_{od}} \sum_{od} \left(\frac{d_{od}^{true} - d_{od}^*}{d_{od}^{true}} \right)^2 \quad (10)$$

$$\%improvement = \frac{MSE_{od}^{historical} - MSE_{od}^{true}}{MSE_{od}^{historical}} \times 100 \quad (11)$$

The obtained results show that O-D correction improvements are more significant as the perturbation value increases. Instead, MSPE of the estimated O-D increases with higher perturbation values for four O-D pairs (20-1, 7-20, 20-7, 7-24), while it is about 6% in average for the remaining O-Ds. As overall result, our model always corrects historical O-D, reaching an average improvement of about 80% in the highest perturbation case.

Figure 3a shows the error at each time interval t of historical and estimated O-D data with the “true” one for O-D pair (1, 20). We can observe the improvement obtained with our dynamic correction model also comparing the estimated flow for O-D (1, 20) with historical and “true” ones (Fig. 3b). Performance results are reported in Table 2 for the three different levels of perturbation previously described.

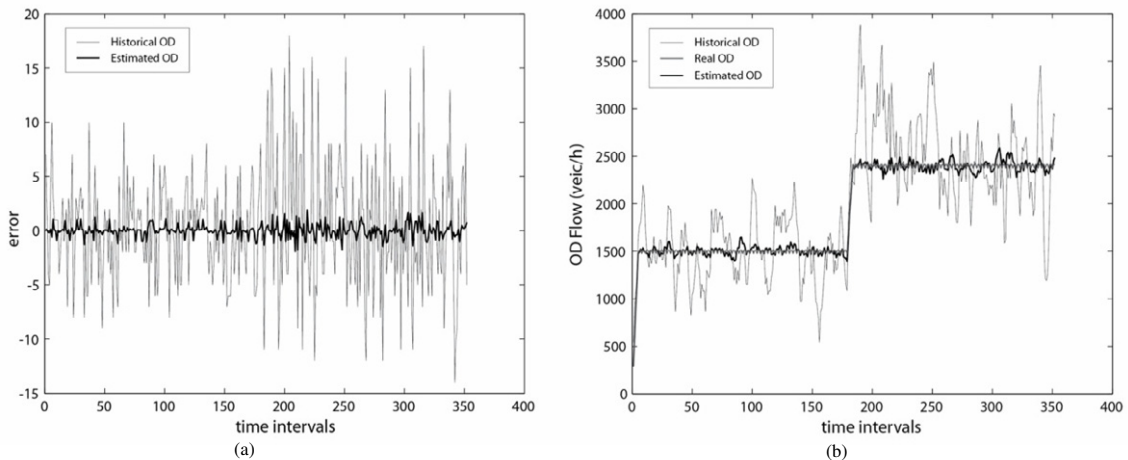


Figure 3. (a) Error at each time interval of historical O-D data vs estimated demand for O-D pair (1, 20); (b) comparison of (1, 20) O-D flows between historical, true and obtained estimated data.

Table 2. Performance results for each O-D at three different levels of perturbation

O - D	MSPE historical/true O-D	MSPE estimated/true O-D	MSE historical/true O-D	MSE estimated/true O-D	% improvement
1 - 20	19,15	5,73	4,46	0,40	91,12
20 - 1	19,64	13,65	4,77	2,46	48,39
1 - 24	20,36	5,49	3,69	0,27	92,80
24 - 1	20,25	5,66	3,63	0,28	92,36
7 - 20	21,67	18,11	3,06	2,23	27,24
20 - 7	20,54	20,48	2,70	2,52	6,96
7 - 24	21,05	18,03	2,79	2,15	23,17
24 - 7	21,13	5,25	2,75	0,21	92,27
1 - 20	48,42	5,61	29,82	0,38	98,72
20 - 1	47,85	29,46	28,84	11,07	61,62
1 - 24	49,92	5,63	22,13	0,30	98,65
24 - 1	47,98	6,03	20,66	0,36	98,27
7 - 20	48,92	34,07	14,89	7,24	51,40
20 - 7	52,80	43,50	15,73	11,36	27,77
7 - 24	51,67	33,64	16,30	7,20	55,82
24 - 7	51,10	6,84	14,58	0,35	97,58
1 - 20	70,78	6,98	64,03	0,54	99,16
20 - 1	69,43	36,68	59,86	17,62	70,57
1 - 24	68,95	6,12	42,89	0,32	99,27
24 - 1	68,60	5,89	41,99	0,33	99,22
7 - 20	66,11	41,18	26,45	10,95	58,59
20 - 7	69,33	53,88	31,87	16,67	47,69
7 - 24	68,86	41,21	28,40	10,43	63,29
24 - 7	71,44	8,57	32,57	0,50	98,48

5. Conclusion

Time-dependent O-D demand matrices are a critical input to dynamic traffic assignment methodology in real-time operational and planning applications. This paper introduces and highlights the potential of Generalized Least Square estimator in conjunction with the BCO-based DTA to estimate the dynamic O-D demand.

We have introduced a DTA algorithm, built around a mesoscopic, time-discrete, Dynamic Network Loading (DNL) model that simulates the operation of a road network. The model is embedded in a new metaheuristic, the Bee Colony Optimization (BCO), which evaluates at each time interval the optimal paths to destination in order to minimize travel times. BCO has been introduced considering the DTA as a high combinatorial problem, when considering all possible paths between O-D pairs. The BCO is capable to solve high level combinatorial

problems with fast convergence performances, allowing to overcome classical demand-flow relationships drawbacks.

The application of the proposed estimator to the Sioux Falls City network has shown the effectiveness of the approach that is able to reduce the bias induced on the considered traffic variables.

Such results have been obtained without using expensive and time consuming data and computing effort, providing a method that should be helpful for practical application when limited resources for data collecting are available. The proposed formulation allows to use, as input data, existing estimates of demand and traffic counts. It also allows to explicitly take into account the analyst's level of confidence (say data reliability) in the assumed starting data by weighting the distances between starting and final estimates using variance matrices.

The numerical analysis has shown that the method is effective with respect to different reliability of historical data and demand values.

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